Euclidian Geometry Tutorial

This tutorial covers the Geometry in Euclid's Elements. Euclid's famous text was "the" book for the study of Geometry until the 19th century. It has been studied by a host of intellectual greats. His systematic approach to Geometry is not a only a tremendous study in how to think and reason, but it became the paradigm that later philosophers would attempt to follow in setting up their own systems of thought. There is really no other mathematical text that rivals its impact on intellectual history.

This tutorial is highly recommended not only for its tremendous historical value, but also as a fine addition to the Geometry-starved Saxon program. The only fault I see with the Saxon programs is its meager treatment of Geometric proofs. Along with Saxon, most modern math texts are downplaying Geometric proofs because they are teaching to the SAT and it does not require proofs. But after going through the magnificent proofs of Euclid, you will see why his work is truly a mathematical classic.

In order to enroll in the second year of the Great Books Tutorial, all students must take my Euclidean Geometry course. The course requires 5-10 hours a week of preparation and should be seen as a college level mathematics course. The Elements comprise the greater part of the freshman year mathematics program at St. John's College and my tutorial is closely modeled after that course.

How does this course compare to a conventional Euclidean Geometry course? Conventional high school Geometry texts study proofs for the first semester of the year and then study the application of Algebra to Geometry that was developed by Descartes (Cartesian) during the second semester. Though based on Euclid, the conventional study of proofs is usually much less thorough than the proofs in Euclid. Cartesian Geometry is on the SAT and should be studied as a separate subject if your child will be taking that quiz. Saxon eliminates a separate year for Geometry entirely and just studies Cartesian Geometry in conjunction with Algebra and Advanced Math.

The text for the tutorial is Euclid's *Elements*, from Green Lion Press. You might also find "The Bones" helpful during your study as well. You can find both books, *The Elements and The Bones,* at Amazon at discount. If you are a Greek student, rather than using the Green Lion edition, you might consider the Fitzpatrick editions which have the Greek text included as well- Volume I,Volume II and Volume III. If you would like a colorized version see the Byrne edition, but please do not use this version for class. To test your knowledge of Euclid after taking this tutorial, please see Euclid: The Game

No grade will be given for the tutorial; however, a numbering system (1-5) will be used to categorize the student's performance. See below for more details.

To see Euclid's elements in the original Greek, click here. You can also find the complete text of Euclid here. You can see the manipulative possibilities of the online text by clicking here. After studying the Elements, if you would like to see the application of Euclid to the study of Astronomy, I recommend studying Euclid's Phaenomena and then Ptolemy's Almagest,

I recommend you download a free demo version or a $10 one year license of Geometer's Sketchpad. Having this software will allow you to make your own versions of the Geometric figures on the computer.

Course Description
Geometry covers an extensive number of proofs from the first nine books of Euclid's Elements. Some of the topics covered are; basic plane geometry, geometric algebra, circles, angles, construction of regular polygons, Eudoxus abstract theory of ratio and proportion, abstract algebra, similar figures and geometric proportions, basic number theory. This course does not address trigonometry or Cartesian geometry. This is an honors level course.

Geometry Tutorial Registration

To register, please print out and mail in our registration form.

Good math links...

[Euclid's Elements]
[How does Geometry apply to cathedral construction?]
[Mathematicians of the Seventeenth and Eighteenth Centuries]
[Web Resources for the History of Mathematics]
[Fibonacci sequence and the Golden Mean]
[Video]

Recommended Supplemental reading

Mathematics- Is God Silent?- James Nickel
[Purchase]

Examples of Common Geometric Abbreviations

The segment AC is equal to the segments BD and FG
AC = BD,FG
Triangle ABC is equal to the square LKJH
^ABC = sq LKJH (or sq KH)
Angle ABC is equal to a right angle
<ABC = ∟
Arc ABC is equal to angle FGH
)ABC = <FGH
Line AC is parallel to line BF
AC || BF
As A is to B so is C to D
A:B::C:D
A is less than, equal to or great than B
A <=> B
The rectangle contained by A,BC is equal to the parallelogram ABCD.
Rect A,BC = ||ogram ABCD
A alike exceeds, falls short of, or equals B as C does of D   A<=>B as C<=>D
Whatever multiple A is of B, that multiple will C be of D. A m B = C m D
The ratio of A to B is equal to that of C to D. A:B::C:D

Sample proposition summary
Book 1, Proposition 5

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.	This is the enunciation of the proposition. It tells you what the proposition will demonstrate and is not included in your summary.
Let ABC be an isosceles triangle having the side AB equal to the side AC, and let the straight lines BD and CE be produced further in a straight line with AB and AC.	This is construction which you need to be able to explain, but is also not included in your summary.
I say that the angle ABC equals the angle ACB, and the angle CBD equals the angle BCE.	The “I say that” tells you once again what the proposition will demonstration and is not included in your proposition.

Take an arbitrary point F on BD. Cut off AG from AE the greater equal to AF the less, and join the straight lines FC and GB.	Here include includes more construction. Once again, be prepared to explain this when you present the proposition, but do not include it in your summary.
Since AF equals AG, and AB equals AC, therefore the two sides FA and AC equal the two sides GA and AB, respectively, and they contain a common angle, the angle FAG.	Here begins the actual proof so you have the first step of your summary. 1. AF = AG 2. AB = AC 3. FA,AC = GA,AB (repeats steps 1&2) 4. <FAG = <FAG (common)
Therefore the base FC equals the base GB, the triangle AFC equals the triangle AGB, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides, that is, the angle ACF equals the angle ABG, and the angle AFC equals the angle AGB.	5. FC = GB 6. ^AFC = ^AGB 7. <ACF = <ABG 8. <AFC = <AGB
Since the whole AF equals the whole AG, and in these AB equals AC, therefore the remainder BF equals the remainder CG.	9. AF = AG (repeat of step 1) 10. AB = AC (repeat of step 2) 11. BF = CG
But FC was also proved equal to GB, therefore the two sides BF and FC equal the two sides CG and GB respectively, and the angle BFC equals the angle CGB, while the base BC is common to them.	12. FC = GB (repeat of 5) 13. BF,FC = CG,GB (repeats 5 and 11) 14. <BFC = <CGB 15. BC=BC
Therefore the triangle BFC also equals the triangle CGB, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.	16. ^BFC = ^CGB
Therefore the angle FBC equals the angle GCB, and the angle BCF equals the angle CBG.	17. <FBC = <GCB 18. <BCF = <CBG
Accordingly, since the whole angle ABG was proved equal to the angle ACF, and in these the angle CBG equals the angle BCF, the remaining angle ABC equals the remaining angle ACB, and they are at the base of the triangle ABC.	19. <ABG = <ACF (repeats step 7) 20. <CBG = <BCF (repeats step 18) 21. <ABC = <ACB
But the angle FBC was also proved equal to the angle GCB, and they are under the base.	22. <FBC = <GCB (repeats step 17)
Therefore *in isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.*	Here Euclid states what the proposition has proven.

The steps you would send into me would be the following- repeated steps may be excluded if you like.

1. AF = AG

2. AB = AC

3. FA,AC = GA,AB

4. <FAG = <FAG

5. FC = GB

6. ^AFC = ^AGB

7. <ACF = <ABG

8. <AFC = <AGB

9. AF = AG

10. AB = AC

11. BF = CG

12. FC = GB

13. BF,FC = CG,GB

14. <BFC = <CGB

15. BC=BC

16. ^BFC = ^CGB

17. <FBC = <GCB

18. <BCF = <CBG

19. <ABG = <ACF

20. <CBG = <BCF

21. <ABC = <ACB

22. <FBC = <GCB

Propositions Covered in Euclid
I all: II all: III all: IV 1-5, 11, 15, 16: V all: VI 1-20, 23, 25,31,33: VII 1-4: VIII 5,11,18: IX 18,20,35,36: X 1,2: XII 1,2,7,10. Prop 1 p. 447: XI 1-4,20,21: XIII 12-15,9,10,16,7,17 Appolonius 1-12

Please send all propositions to gbt@gbt.org by Sunday night. The subject line should be in the following format. John Smith 4.2-4.4 Geoprop. Please do not send as attachments, but in the body of your email.

Geometry weekly assignments

Week #	8-10am Monday	10-12am Monday	8-10am Tuesday
1	**Book 1 Read Definitions, Common notions, Postulates and Propositions 1 & 3**	**Book 1 Read Definitions, Common notions, Postulates and Propositions 1 & 2**	**Book 1 Read Definitions, Common notions, Postulates and Propositions 1 & 2**

2	**Book 1 Read Definitions, Common notions, Postulates and Propositions 1-3**	**Book 1 Read Definitions, Common notions, Postulates and Propositions 1-3**	**Book 1 Read Definitions, Common notions, Postulates and Propositions 1-3**
3	**1.2-7**	**1.2-7**	**1.2-7**
4	**1.6-15**	**1.6-15**	**1.6-15**
5	**1.10-18**	**1.10-18**	**1.10-18**
6	**1.15-22**	**1.16-23**	**1.18-27**
7	**1.20-27**	**1.21-28**	**1.25-32**
8	**1.25-33**	**1.26-34**	**1.29-37**
9	**1.30-37**	**1.31-38**	**1.37-46**
10	**1.37-46**	**1.38-47**	**1.43-48**
11	**1.45-2.2**	**1.45-2.3**	**1.47-2.6**
12	1.47-2.5	1.47-2.6	**2.2-2.7**
13	**2.2-2.7**	**2.2-2.7**	**2.6-12**
14	**2.6-10**	**2.6-11**	**2.9-14**
15	**2.9-14**	**2.8-14**	**2.11-14 3.1-2**
16	2.12-14 3.1-2	2.10-14 3.1	**2.14 3.1-4**
17	3.1-6	2.12,13,14 3.1,2	3.4-10
18	3.7-12	3.3-9	3.9-16
19	3.12-19	3.8-15	3.15-25
20	3.15-22	3.14-21	**3.22-31**
21	3.19-27	3.17-23	3.30-36
22	3.26-33	3.24-32	3.35-37 4.1-3
23	3.33-37 4.1,2	3.32-37, 4.1-2	4.4,5,11,15,16
24	**3.37 4.1-5**	**3.37 4.1-5**	**4.16, 5.1-4**
25	**IV 5, 11, 15, 16: V 1**	**IV 5, 11, 15, 16: V 1**	**5.3-9**
26	**5.1-6**	**5.1-6**	**5.6-11**
27	**V 3-9**	**V 3-9**	**5.9-15**
28			**5.15-22**

	V 6-10	**V 6-10**	
29	**V 11-17**	**V 11-17**	**5.19-25**
30	**V 17-23**	**V 18-24**	**VI.1-6**
31	**V 22-25 VI 1-3**	**V 22-25 VI 1-3**	**VI.5-11**
32	**V 22-25 VI 1-3**	**6*4-12**	**VI.9-17**
33			
34	VI 6-13	VI 7-13	6. 17-20,23,25,31,33
35	Plato's Meno	Plato's Meno	Plato's Meno

Grading Performance in Geometry

Geometry students will not be given an evaluation or grade for their work in the tutorial. In order to assess your work, you will need to keep a record of your performance on each proposition that the tutor calls on you to demonstrate in class. There are four levels of performance.

Explain the steps of your proposition-

1. **From memory (You cannot miss more than 2 steps)**
2. **From written notes**
3. **From the text**
4. **Unable to explain steps**
5. **Did not have steps prepared**

Please keep a record of your performance on all propositions that you demonstrated so that you can find your semester average.

Course Description

The Euclidean Geometry tutorial is an extensive survey of the proofs contained in Euclid's Elements. The tutorial is based on the original Element's rather than a summary text. The proofs covered deal with such subjects as;

The fundamentals of geometry: theories of triangles, parallels, and area.
Geometric algebra.
Theory of circles.
Constructions for inscribed and circumscribed figures.
Theory of abstract proportions.
Similar figures and proportions in geometry.
Fundamentals of number theory.
Continued proportions in number theory.
Number theory.
Classification of incommensurables.
Measurement of figures.
Regular solids.

Quotes on Euclid from History

From ABRAHAM LINCOLN by G. Frederick Owen
"He (Lincoln) read in the fields of politics, literature, philosophy, and science. But most of all he 'read law'. In the course of that law-reading, he says:
' I constantly came upon the word demonstrate. I thought at first that I understood its meaning, but soon became satisfied that I did not. I said to myself, 'What do I mean when I demonstrate more than when I reason or prove?'. I consulted Webster's Dictionary. That told of 'certain proof', 'proof beyond the possibility of doubt' ; but I could form no idea what of

sort of proof that was. I thought that a great many things were proved beyond a possibility of doubt, without recourse to any such extraordinary process of reasoning as I understood 'demonstration' to be. I consulted all the dictionaries and books of reference I could find, but with no better results. You might as well have defined 'blue' to a blind man. At last I said, 'Lincoln, you can never make a lawyer if you do not understand what 'demonstrate' means ; and I left my situation in Springfield, went home to my father's house, and stayed there until I could give any proposition in the six books of Euclid at sight. I then found out what 'demonstrate' means, and went back to my law studies.'
Abraham Lincoln

"At noon we went home for dinner and then back again for history in the afternoon. The history was a pretty hard paper and I got dreadfully mixed up in the dates. Still, I think I did fairly well today. But oh, Diana, tomorrow the geometry exam comes off and when I think of it it takes every bit of determination I possess to keep from opening my **Euclid**. If I thought the multiplication table would help me any I would recite it from now till tomorrow morning."

Anne of Green Gables

"if God exists and if He really did create the world, then, as we all know, He created it according to the geometry of Euclidand the human mind with the conception of only three dimensions in space. Yet there have been and still are geometricians and philosophers, and even some of the most distinguished, who doubt whether the whole universe, or to speak more widely, the whole of being, was only created in **Euclid's geometry**; they even dare to dream that two parallel lines, which according to **Euclid** can never meet on earth, may meet somewhere in infinity. [Lobechevski] I have come to the conclusion that, since I can't understand even that, I can't expect to understand about God."

Brothers Karamazov

"On my return to Geneva, I passed two or three years at my uncle's, expecting the determination of my friends respecting my future establishment. His own son being devoted to engineering, was taught drawing, and instructed by his father in the elements of **Euclid:** I partook of these instructions, but was principally fond of drawing."

The Confessions of Jean-Jacques Rousseau

Then when a little more I raised my brow,
I spied the master of the sapient throng,
Seated amid the philosophic train.
Him all admire, all pay him reverence due.
There Socrates and Plato both I mark'd
Nearest to him in rank, Democritus,
Who sets the world at chance, Diogenes,
With Heraclitus, and Empedocles,
And Anaxagoras, and Thales sage,
Zeno, and Dioscorides well read
In nature's secret lore. Orpheus I mark'd
And Linus, Tully and moral Seneca,
Euclid and Ptolemy, Hippocrates,
Galenus, Avicen, and him who made
The commentary vast, Averroes.

The Divine Comedy (Inferno)

How is it, then, with the whale? True, both his eyes, in themselves, must simultaneously act; but is his brain so much more comprehensive, combining, and subtle than man's, that he can at the same moment of time attentively examine two distinct prospects, one on one side of him, and the other in an exactly opposite direction? If he can, then is it as marvellous a thing in him, as if a man were able simultaneously to go through the demonstrations of two distinct problems in **Euclid**. Nor, strictly investigated, is there any incongruity in this comparison.

Moby Dick

I WILL give no more of the details of my hero's earlier years. Enough that he struggled through them, and at twelve years old knew every page of his Latin and Greek Grammars by heart. He had read the greater part of Virgil, Horace, and Livy, and I do not know how many Greek plays: he was proficient in arithmetic, knew the first four books of **Euclid** thoroughly, and had a fair knowledge of French. It was now time he went to school, and to school he was accordingly to go, under the famous Dr. Skinner of Roughborough**.**

Way of All Flesh- Samuel Butler

Euclid alone has looked on Beauty bare.

Edna St. Vincent Millay

When I am violently beset with temptations, or cannot rid myself of evil thoughts, [I resolve] to do some Arithmetic, or Geometry, or some other study, which necessarily engages all my thoughts, and unavoidably keeps them from wandering.

Edwards, Jonathan

> I picked up the dignified-looking book called Euclid's <u>Elements</u> for the first time as a 15-year-old sophomore with a passionate dislike of math. Although the black hardcover book with its silky white pages had a striking visage unlike any math textbook I had previously encountered, I was not going to let appearances deceive me. I knew math was math, and no pretty disguise would change that. My evaluation of Euclid was dramatically altered from the first page. Euclidimpressed me with three primary items that had a significant impact on my thinking and greatly prepared me for future studies.
>
> First, his method of intellectual organization surprised and delighted me. He did not confuse his point with pointless palaver or confounding circumlocutions. He knew exactly what he wanted to prove, why he wanted to prove it, how he wanted to prove it, and he proved it! I had always valued organization in real life, but I had never thought about intellectual organization before. **Euclid** presented a picture of what mental organization looked like, and how beautifully intricate it could be. "Perhaps," I began to consider, "this structure and clarity could be applied to other intellectual pursuits, like writing, speaking, and heavens, maybe even Algebra." **Euclid** made me aware of the beauty of orderly thought and inspired me to seek and create that structure elsewhere.
>
> Secondly, the Elements introduced a cast of abstract ideas that forced me to exercise my "mind's eye" and greatly aided my comprehension of other abstract ideas. For instance, discovering that every "line" (based on **Euclid**'s definition) was merely a representation of a real line, which could not be reproduced physically, helped clarify what Plato meant by his theory of the forms. I was fascinated by the almost mystic quality of "points," "triangles" and the rest, and contemplating them created a new "cabinet" in my mind where I could file away related information of an abstract nature.
>
> The third thing I loved about **Euclid** was his method of building irrefutable arguments. By constructing simple proofs from undeniable definitions and "common notions" and then using his simple proofs to prove complex ones, he assembles a venerable army of unassailable arguments. Often I would read the thesis of a complex proof and think, "Oh wow! He's never going to be able to convince me of this ," but in the end, I was always forced to concede that what he said was true. As a somewhat over-confident youth, these intellectual defeats surprised me at first, then humbled me, and finally thrilled me.
>
> I fell in love with geometric syllogism, because of its organization, abstract content, and dazzling certainty. Studying **Euclid** encouraged me to grasp these principles for my own, and enriched my reading of philosophers who have striven for clarity in these areas as well. Euclid was also vital stepping stone to reading Kant, who was the greaquiz academic challenge I have faced.
>
> **From Kate Peacock's (former Great Books V student) admissions essay**

- Plato

[Geometry graphic](#)
[Rozeta Pary notre-dame chalger](#)

Ancient and Medieval art is often constructed using the geometric forms found in Euclid.

[God the Geometer](#)

"God always Geometrizes"
- Plato

Here are some more links for the connections between Geometry and Art

Wood carving
Quilt Patterns
ETS GBT V boxes: 1 2 3
Corinthian Rondel
Greek Geometric Pottery
Windows of Notre Dame
Leonardo's Geometric Man